Study of the parity of even and odd numbers

Maxime Luce

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Abstract

We will prove that for any natural integer n, if it is even, then its square is also even. In the same way, we will prove that for any natural whole n, if it is odd, its square is also odd.

Theorem 1. *If n is even, then n^2 is also even with $n \in \mathbb{N}$.*

Proof. Let $(k, n) \in \mathbb{N}^2, n = 2k$

We can now compute n^2.

$$n^2 = (2k)^2 \tag{1}$$
$$= 4k^2 \tag{2}$$
$$= 2(2k^2) \tag{3}$$

Let $k' = 2k^2$. Then, we get:

$$n^2 = 2k' \tag{4}$$

As we can write n^2 as $2k'$ with $k' \in \mathbb{Z}$, it is also even. □

Theorem 2. *If n is odd, then n^2 is also odd with $n \in \mathbb{N}$.*

Proof. Let $(k, n) \in \mathbb{N}^2, n = 2k + 1$

We can now compute n^2.

$$n^2 = (2k + 1)^2 \tag{5}$$
$$= 4k^2 + 4k + 1 \tag{6}$$
$$= 2(2k^2 + 2k) + 1 \tag{7}$$

Let $k' = 2k^2 + 2k$. Then, we get:

$$n^2 = 2k' + 1 \tag{8}$$

As we can write n^2 as $2k' + 1$ with $k' \in \mathbb{Z}$, it is also odd. □